Optimal Sensor Placement for Source Extraction

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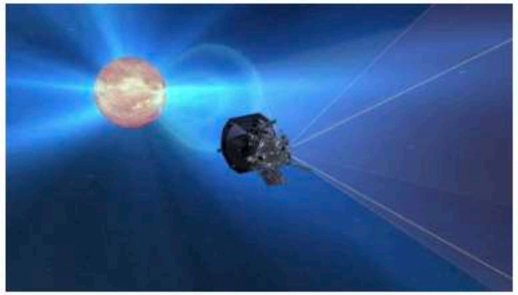
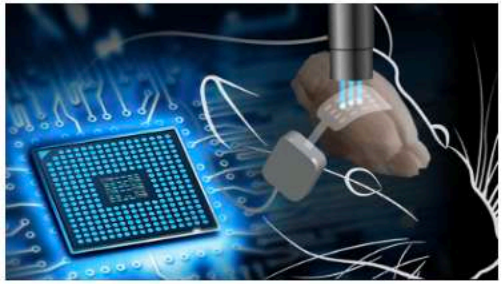
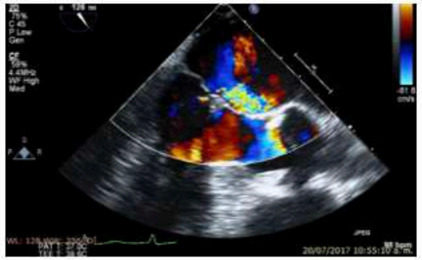
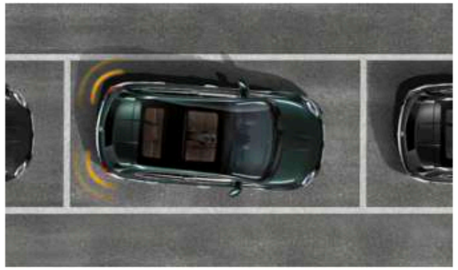
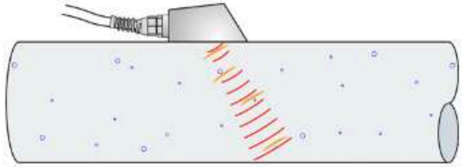
Motivation

Sensors are being used in a variety of domains:

- Industry
- Medicine
- Wireless communications
- Aerospace engineering
- Biomedical engineering
- Civil engineering
- Environmental study
- Robotics
- …



Motivation

More sensors ⇨ More information ⇨ Improving the performance

❑ Limiting the number of sensors

- **Economical interest**
- **Energy**: reducing the required energy for the power supply
- **Weight**
- **Reducing computational complexity**
- **Ergonomic design and arrangement** e.g. motion capture

...

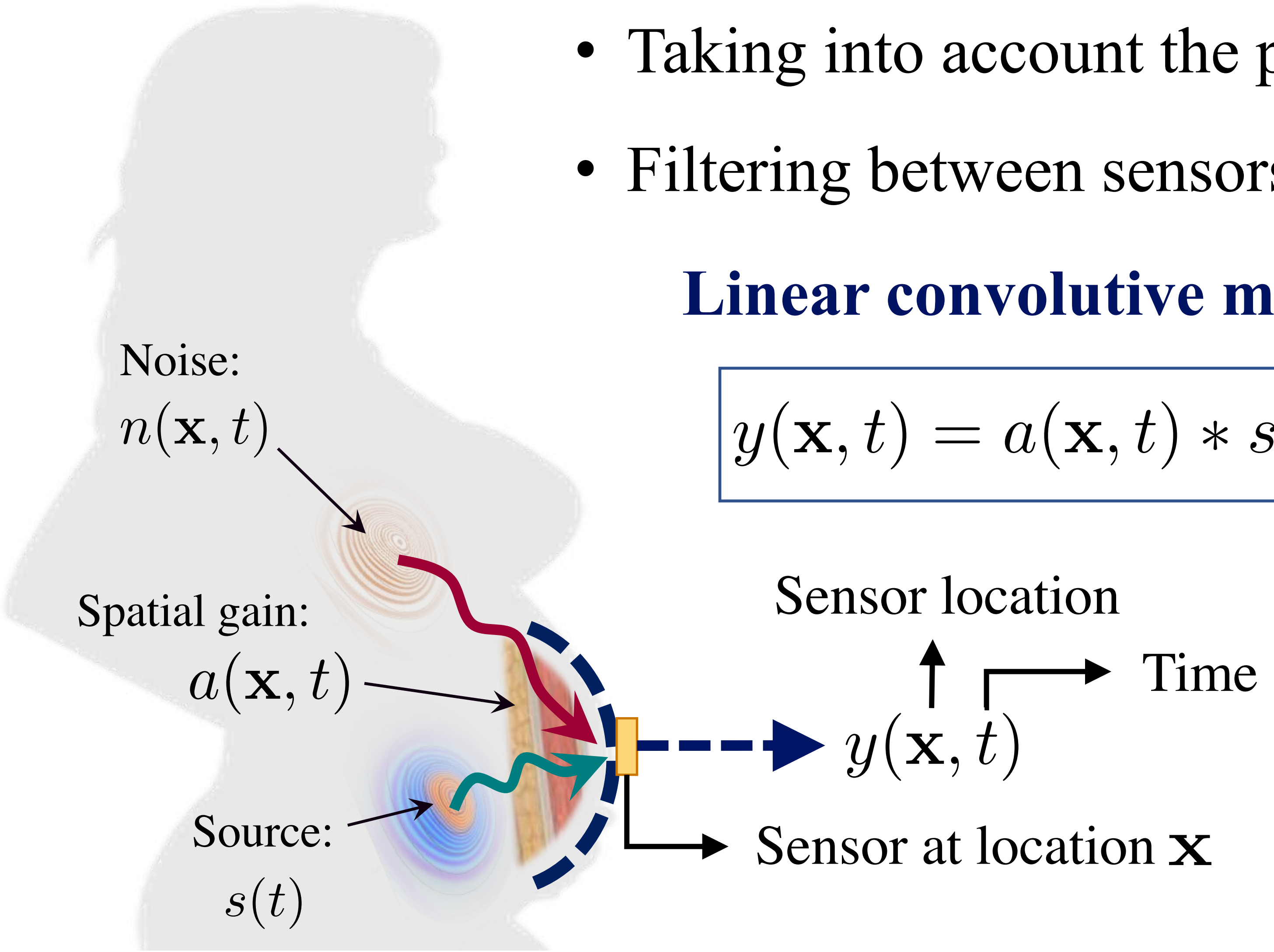
Optimal sensor placement is important to collect the best data!

Optimal sensor placement for source extraction

- Acoustic signals *e.g.* PCG
- Taking into account the propagation delay
- Filtering between sensors and sources

Linear convolutive mixture model:

$$\boxed{y(\mathbf{x},t) = a(\mathbf{x},t) * s(t) + n(\mathbf{x},t)}$$

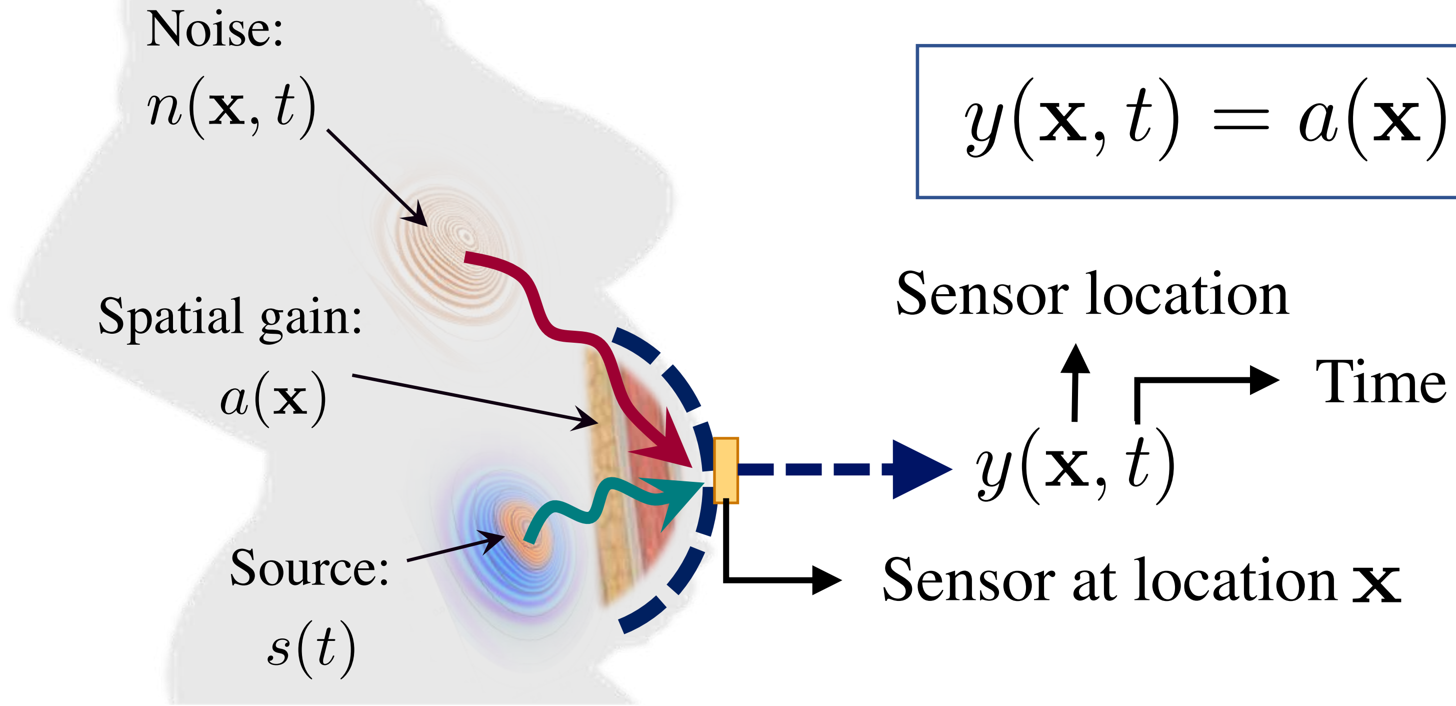


Optimal sensor placement for source extraction

- Electrical signals *e.g.* ECG, EEG, MEG
- Quasi-static approximation of Maxwell law
- Neglect the propagation times (fast propagation)

Time-invariant linear instantaneous model:

$$\boxed{y(\mathbf{x}, t) = a(\mathbf{x})s(t) + n(\mathbf{x}, t)}$$



Optimal sensor placement for source extraction

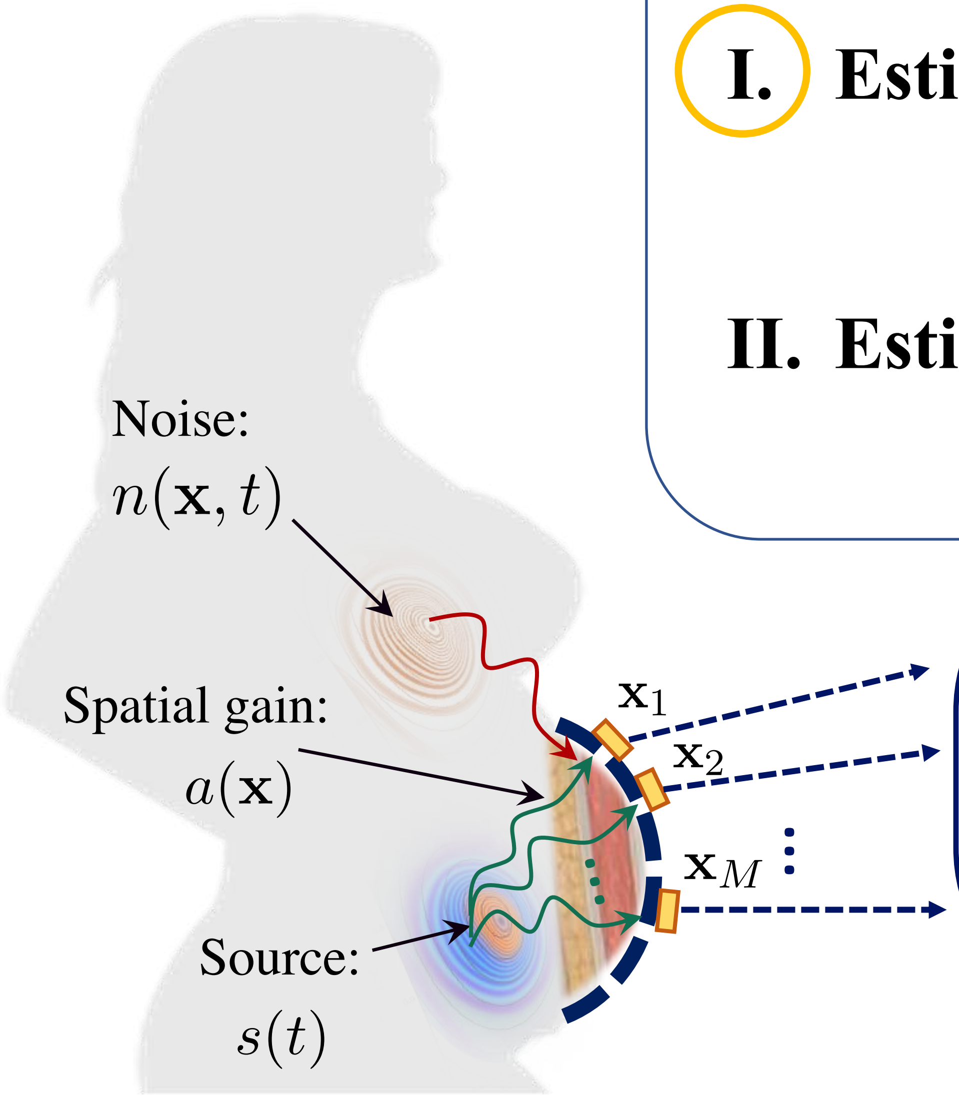
Objectives to be achieved

I. Estimating the source signal $s(t)$:

→ source extraction problem

II. Estimating the spatial gain $a(\mathbf{x})$ for all $\mathbf{x}$:

→ spatial interpolation *e.g.* kriging

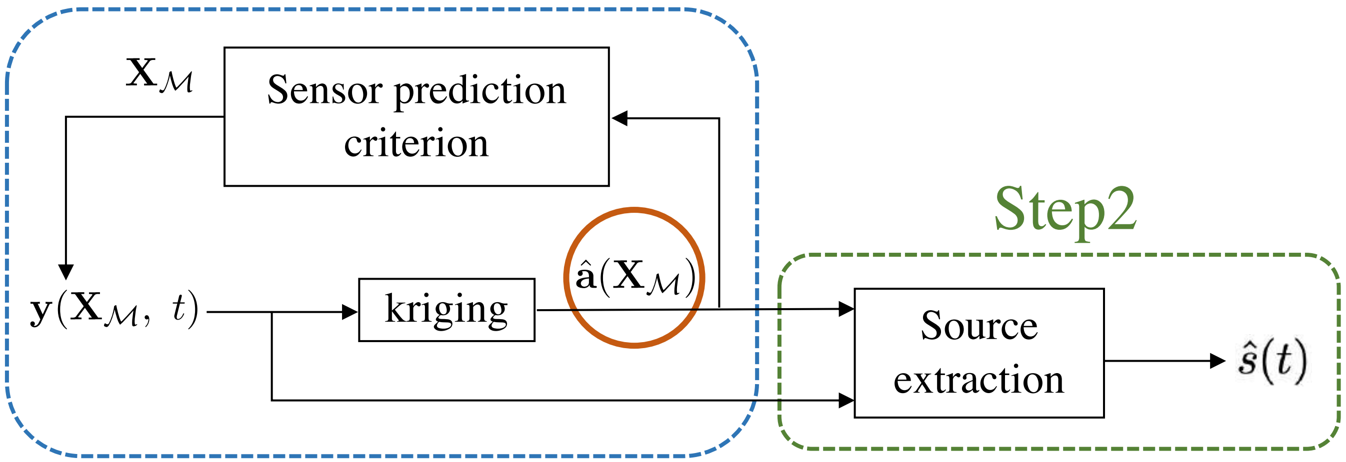


$$\begin{pmatrix} y(\mathbf{x}_1, t) = a(\mathbf{x}_1)s(t) + n(\mathbf{x}_1, t) \\ y(\mathbf{x}_2, t) = a(\mathbf{x}_2)s(t) + n(\mathbf{x}_2, t) \\ \vdots \\ y(\mathbf{x}_M, t) = a(\mathbf{x}_M)s(t) + n(\mathbf{x}_M, t) \end{pmatrix} = \mathbf{y}(\mathbf{X}_{\mathcal{M}}, t)$$

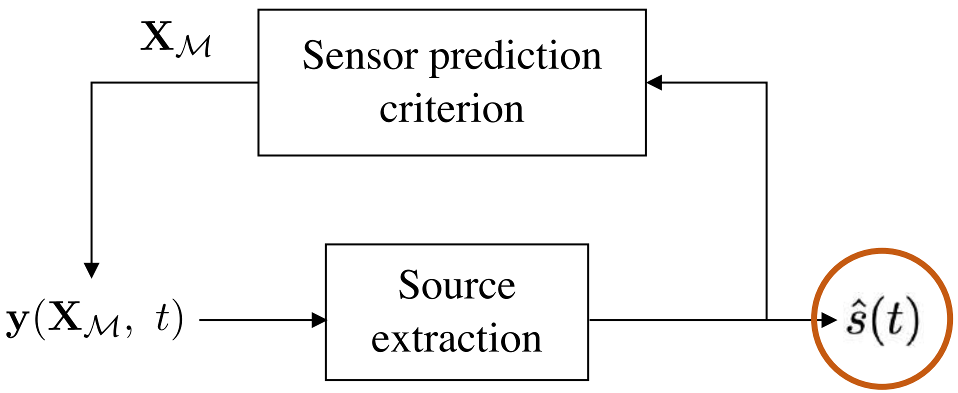
Classical kriging approaches *v.s.* our approach

$$\mathbf{y}(\mathbf{X}_{\mathcal{M}}, t) = \mathbf{a}(\mathbf{X}_{\mathcal{M}})s(t) + \mathbf{n}(\mathbf{X}_{\mathcal{M}}, t)$$

Step1



(a) Kriging approach: estimation spatial gain [1, 2]



(b) Our approach: estimating the source

Criterion:
- Mutual information
- Entropy

Criterion:
- To be presented …

[1] M. C. Shewry , '*Maximum entropy sampling*', J. of Applied Statistics , (1987).

[2] N. Cressie, '*The origins of kriging*', Mathematical Geology, (1990).

Contributions to be Presented

I. **Criterion:** Robust sensor placement for signal extraction

II. **Optimization:** Gradient-based algorithm with spatial regularization

Contributions

I. **Criterion:** Robust sensor placement for signal extraction

II. **Optimization:** Gradient-based algorithm with spatial regularization

SNR as a criterion for optimal sensor placement

- Model of recordings: $\mathbf{y}(\mathbf{X}_{\mathcal{M}}, t) = \mathbf{a}(\mathbf{X}_{\mathcal{M}})s(t) + \mathbf{n}(\mathbf{X}_{\mathcal{M}}, t) \Longrightarrow \hat{s}(t)$

- Estimation of $s(t)$: Linear source extraction

$$\hat{s}(t) = \underbrace{\mathbf{f}(\mathbf{X}_{\mathcal{M}})}_{\text{Extractor vector}}{}^T \mathbf{y}(\mathbf{X}_{\mathcal{M}}, t) = \underbrace{\mathbf{f}(\mathbf{X}_{\mathcal{M}})^T \mathbf{a}(\mathbf{X}_{\mathcal{M}})s(t)}_{\text{Signal}} + \underbrace{\mathbf{f}(\mathbf{X}_{\mathcal{M}})^T \mathbf{n}(\mathbf{X}_{\mathcal{M}}, t)}_{\text{Noise}}$$

- Estimation of the extraction vector $\mathbf{f}$: maximizing the output SNR

$$\mathrm{SNR}(\mathbf{f}(\mathbf{X}_{\mathcal{M}})) = \frac{\mathbb{E}_t\left[\left(\mathbf{f}(\mathbf{X}_{\mathcal{M}})^T \mathbf{a}(\mathbf{X}_{\mathcal{M}})s(t)\right)^2\right]}{\mathbb{E}_t\left[\left(\mathbf{f}(\mathbf{X}_{\mathcal{M}})^T \mathbf{n}(\mathbf{X}_{\mathcal{M}}, t)\right)^2\right]} = \underbrace{\sigma_s^2}_{\text{Source variance}} \frac{\mathbf{f}(\mathbf{X}_{\mathcal{M}})^T \mathbf{a}(\mathbf{X}_{\mathcal{M}})\mathbf{a}(\mathbf{X}_{\mathcal{M}})^T \mathbf{f}(\mathbf{X}_{\mathcal{M}})}{\mathbf{f}(\mathbf{X}_{\mathcal{M}})^T \underbrace{\mathbf{C}^n(\mathbf{X}_{\mathcal{M}}, \mathbf{X}_{\mathcal{M}})}_{\text{Noise variance}} \mathbf{f}(\mathbf{X}_{\mathcal{M}})}$$

SNR as a criterion for optimal sensor placement

- Maximizing the SNR: $\mathbf{f}^*(\mathbf{X}_\mathcal{M}) = \mathbf{C}^n(\mathbf{X}_\mathcal{M}, \mathbf{X}_\mathcal{M})^{-1}\mathbf{a}(\mathbf{X}_\mathcal{M})$

$$\mathrm{SNR}(\mathbf{f}^*(\mathbf{X}_\mathcal{M})) = \sigma_s^2 \, \overbrace{\underbrace{\mathbf{a}(\mathbf{X}_\mathcal{M})^T}\,\underbrace{\mathbf{C}^n(\mathbf{X}_\mathcal{M}, \mathbf{X}_\mathcal{M})^{-1}}_{\text{Known}}\,\underbrace{\mathbf{a}(\mathbf{X}_\mathcal{M})}}^{\text{GP}} = J(\mathbf{X}_\mathcal{M})$$

- Where to put the sensors? $\mathbf{X}^*_\mathcal{M} = \arg\max_{\mathbf{X}_\mathcal{M}} J(\mathbf{X}_\mathcal{M})$

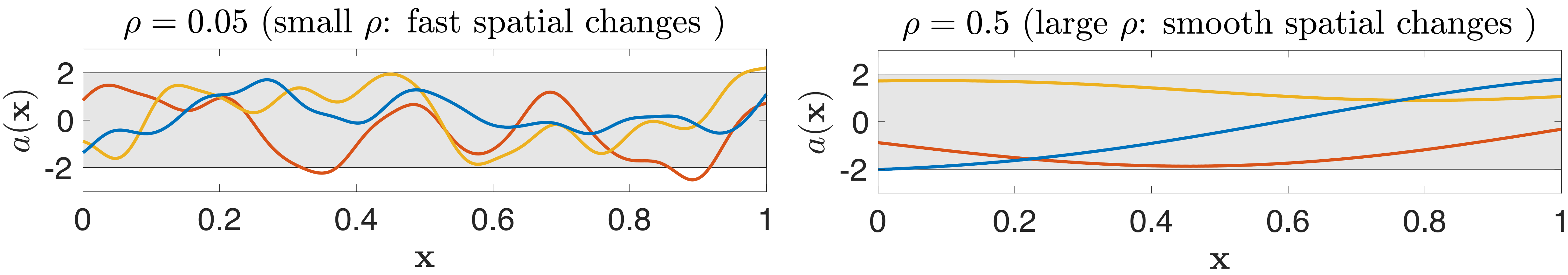
- Gaussian Process assumption: $a(\mathbf{x}) \sim \mathcal{GP}\big(\underbrace{m^a(\mathbf{x})}_{\text{Prior}}, \underbrace{k^a(\mathbf{x}, \mathbf{x}')}_{\text{Uncertainty}}\big)$

Why Gaussian Process?

$$a(\mathbf{x}) \sim \mathcal{GP}\big(m^a(\mathbf{x}), k^a(\mathbf{x}, \mathbf{x}')\big)$$

- Hyperparameters: representing signal properties *e.g.* magnitude and smoothness

$$\boxed{m^a(\mathbf{x}) = 0 \quad , \quad k^a(\mathbf{x}, \mathbf{x}') = \sigma^2 \exp(-(\mathbf{x} - \mathbf{x}')^2/(2\rho^2))}$$



- Representing many shapes

- Easy to compute various quantities *e.g.* marginal/conditional distributions

Probability density function (pdf) of the output SNR

Output SNR: $$\frac{1}{\sigma_s^2}\,\mathrm{SNR}(\boldsymbol{f}^*(\mathbf{X}_\mathcal{M})) = \mathbf{a}(\mathbf{X}_\mathcal{M})^T \mathbf{C}^n(\mathbf{X}_\mathcal{M},\mathbf{X}_\mathcal{M})^{-1}\mathbf{a}(\mathbf{X}_\mathcal{M})$$

Random $\mathbf{a}(\mathbf{X}_\mathcal{M})$ ⟹ Random SNR

$$\mathsf{w}(\mathbf{X}_\mathcal{M}) \triangleq \frac{1}{\sigma_s^2}\,\mathrm{SNR}(\boldsymbol{f}^*(\mathbf{X}_\mathcal{M}))$$

$$\mathsf{w}(\boldsymbol{X}_\mathcal{M}) = \sum_{i=1}^{M} d_i \mathsf{v}_i$$

- Noncentral chi-squared distribution
- Independent random variables

Distribution of the SNR: $g_\mathsf{w}(w)$ ⟹ Sensor placement criterion

[1] F. Ghayem, B. Rivet, C. Jutten, R. C. Farias, '*Robust Sensor Placement for Signal Extraction*', IEEE Trans. on Signal Processing (submitted)

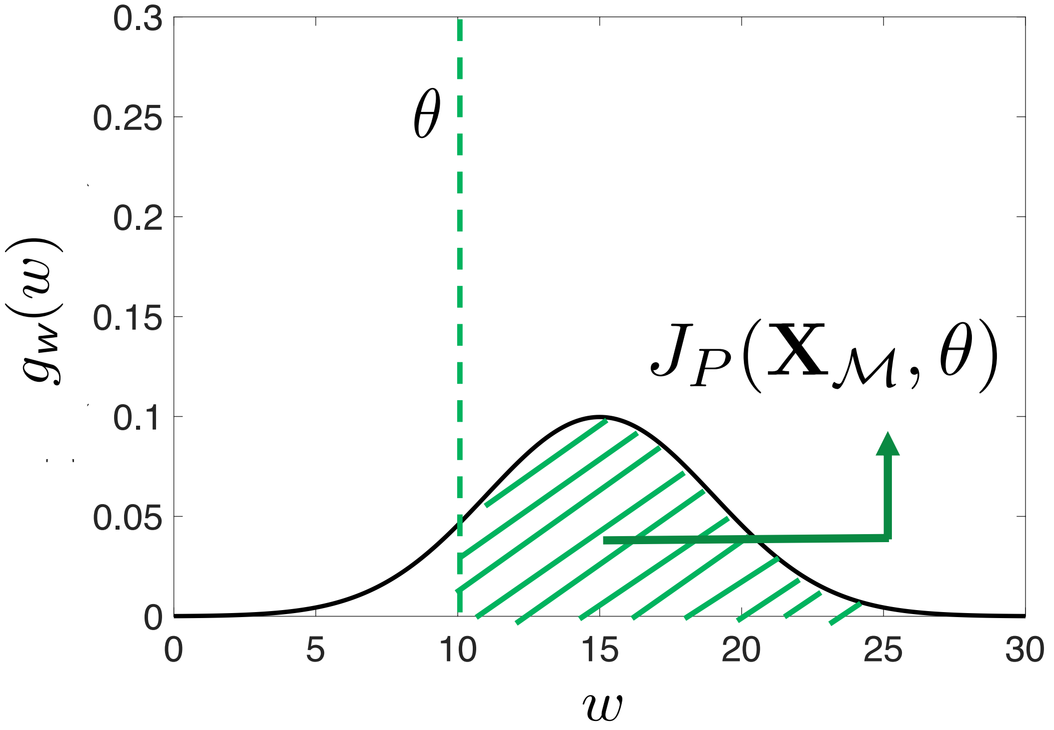
Robust sensor placement criterion for signal extraction

Output SNR: $J(\mathbf{X}_{\mathcal{M}}) = \mathbf{a}(\mathbf{X}_{\mathcal{M}})^T \mathbf{C}^n(\mathbf{X}_{\mathcal{M}}, \mathbf{X}_{\mathcal{M}})^{-1} \mathbf{a}(\mathbf{X}_{\mathcal{M}})$

Target function: $\boxed{J_P(\mathbf{X}_{\mathcal{M}}, \theta) = Pr(w(\mathbf{X}_{\mathcal{M}}) > \theta) = 1 - G_w(\theta)}$

J_P → probability

G_w → cdf of $w(\mathbf{X}_{\mathcal{M}})$



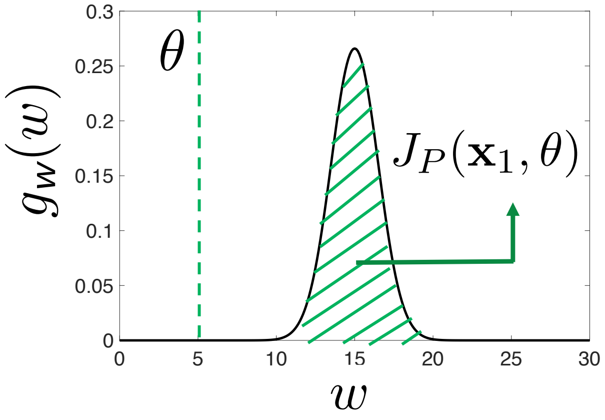
$$\hat{\mathbf{X}}_{\mathcal{M}} = \underset{\mathbf{X}_{\mathcal{M}}}{\operatorname{argmax}}\, J_P(\mathbf{X}_{\mathcal{M}}, \underset{?}{\underline{\theta}})$$

[*Robust Sensor Placement for Signal Extraction*', F. Ghayem, B. Rivet, C. Jutten, R. C. Farias, transactions on signal processing (submitted)]

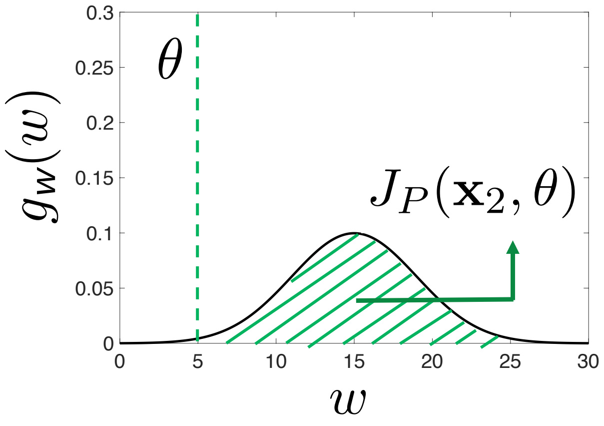
Effect of parameter θ

Target function: $$J_P(\mathbf{X}_{\mathcal{M}}, \theta) = Pr(w(\mathbf{X}_{\mathcal{M}}) > \theta)$$

Case1: small θ



at $\mathbf{x}_1$: small variance



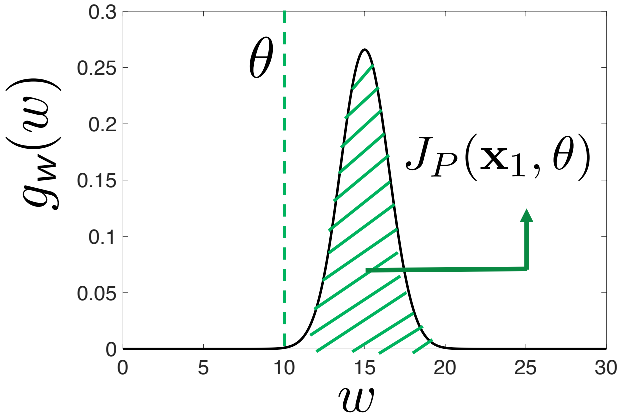
at $\mathbf{x}_2$: large variance

$(\theta < \theta_{median})$

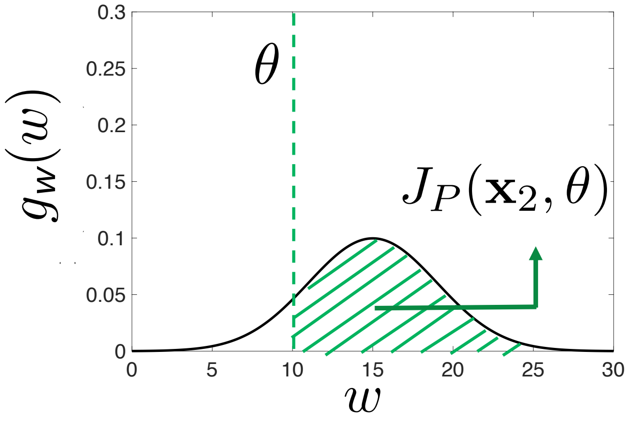
$$J_P(\mathbf{x}_1, \theta) \cong J_P(\mathbf{x}_2, \theta) \cong 1$$

⇒ Non-discriminative

Case2: large θ



at $\mathbf{x}_1$: small variance



at $\mathbf{x}_2$: large variance

$(\theta < \theta_{median})$

$$J_P(\mathbf{x}_1, \theta) \quad > \quad J_P(\mathbf{x}_2, \theta)$$

⇒ Discriminative

Discrete *v.s.* Continuous Optimization

$$\hat{\mathbf{X}}_{\mathcal{M}} = \underset{\mathbf{X}_{\mathcal{M}}}{\operatorname{argmax}}\, J_P(\mathbf{X}_{\mathcal{M}}, \theta) \qquad J_P(\mathbf{X}_{\mathcal{M}}, \theta) = Pr(w(\mathbf{X}_{\mathcal{M}}) > \theta)$$

Optimization over a continuous spatial space (off-the-grid)

Non-convex!

Sensitive to the initialization:

Bad initialization $\Longrightarrow$ Bad local minima/maxima

Discrete spatial space framework (on-the-grid)

Combinatorial search!

- Performance is limited by the grid size

Increasing the grid size $\Longrightarrow$

- High computational complexity
- Multiple closely spaced sensors

Greedy approach for optimal sensor placement

- Greedy approach: sequentially selecting $N < M$ sensors at a time
  - Already allocated sensors at $\mathbf{X}_{\mathcal{K}} = [\mathbf{x}_1,\ \mathbf{x}_2, \ldots,\ \mathbf{x}_K]$
  - Choosing N other sensors' locations $\mathbf{X}_{\mathcal{N}} = [\mathbf{x}_{K+1},\ \mathbf{x}_{K+2}, \ldots,\ \mathbf{x}_{K+N}]$

$\mathbf{X}_{\mathcal{M}}$: concatenation of $\mathbf{X}_{\mathcal{K}}$ and $\mathbf{X}_{\mathcal{N}}$ $\Longrightarrow$ $\mathbf{X}_{\mathcal{M}} = \begin{bmatrix} \mathbf{X}_{\mathcal{K}} \\ \mathbf{X}_{\mathcal{N}} \end{bmatrix}$

$$J_P(\mathbf{X}_{\mathcal{N}}, \theta | \mathbf{X}_{\mathcal{K}}) = \Pr(w(\mathbf{X}_{\mathcal{N}} | \mathbf{X}_{\mathcal{K}}) > \theta) = 1 - G_{w(\mathbf{X}_{\mathcal{N}} | \mathbf{X}_{\mathcal{K}})}(\theta)$$

$$\boxed{\hat{\mathbf{X}}_{\mathcal{N}} = \arg\max_{\mathbf{X}_{\mathcal{N}}} J_P(\mathbf{X}_{\mathcal{N}}, \theta | \mathbf{X}_{\mathcal{K}})}$$

Estimation of the spatial gain

$$\mathbf{X}_{\mathcal{M}} \longrightarrow \mathbf{f}^*(\mathbf{X}_{\mathcal{M}}) = \mathbf{C}^n(\mathbf{X}_{\mathcal{M}}, \mathbf{X}_{\mathcal{M}})^{-1} \boxed{\mathbf{a}^*(\mathbf{X}_{\mathcal{M}})} \longrightarrow \hat{s}(t)$$

$\mathbf{a}^*(\mathbf{X}_{\mathcal{M}})$: true value of the spatial gain $\longrightarrow$?

$\hat{\mathbf{a}}(\mathbf{X}_{\mathcal{M}})$: estimated value of the spatial gain

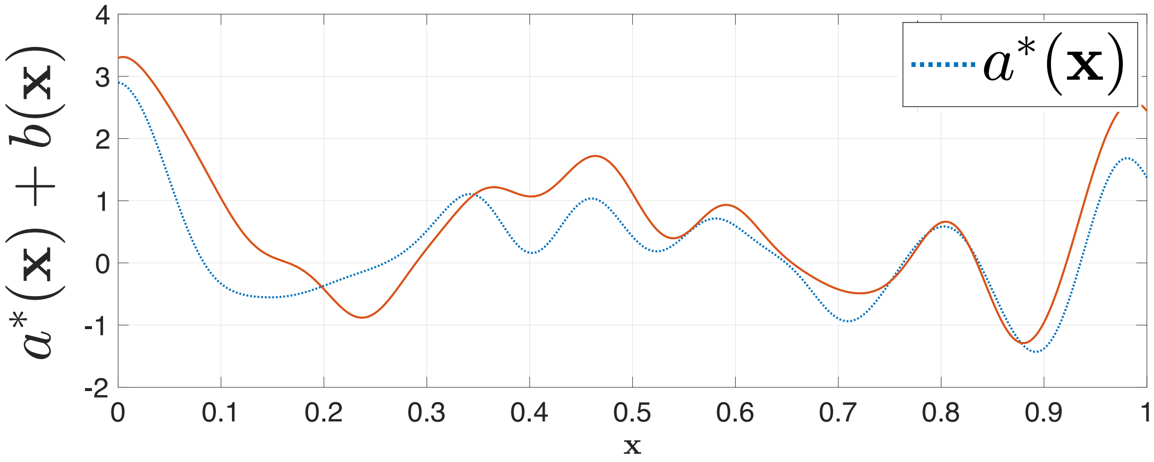
$$a(\mathbf{x}) \sim \mathcal{GP}(\underbrace{m^a(\mathbf{x})}_{\text{Prior}}, \underbrace{k^a(\mathbf{x}, \mathbf{x}')}_{\text{Uncertainty}}) \Longrightarrow \hat{a}(\mathbf{x}) = \underbrace{m^a(\mathbf{x})}_{\text{Deterministic}} + \underbrace{u(\mathbf{x})}_{\text{Stochastic}}$$

Estimation of the spatial gain

$$a(\mathbf{x}) \sim \mathcal{GP}(\underbrace{m^a(\mathbf{x})}_{\text{Prior}}, \underbrace{k^a(\mathbf{x}, \mathbf{x}')}_{\text{Uncertainty}}) \Longrightarrow \hat{a}(\mathbf{x}) = \underbrace{m^a(\mathbf{x})}_{\text{Deterministic}} + \underbrace{u(\mathbf{x})}_{\text{Stochastic}}$$

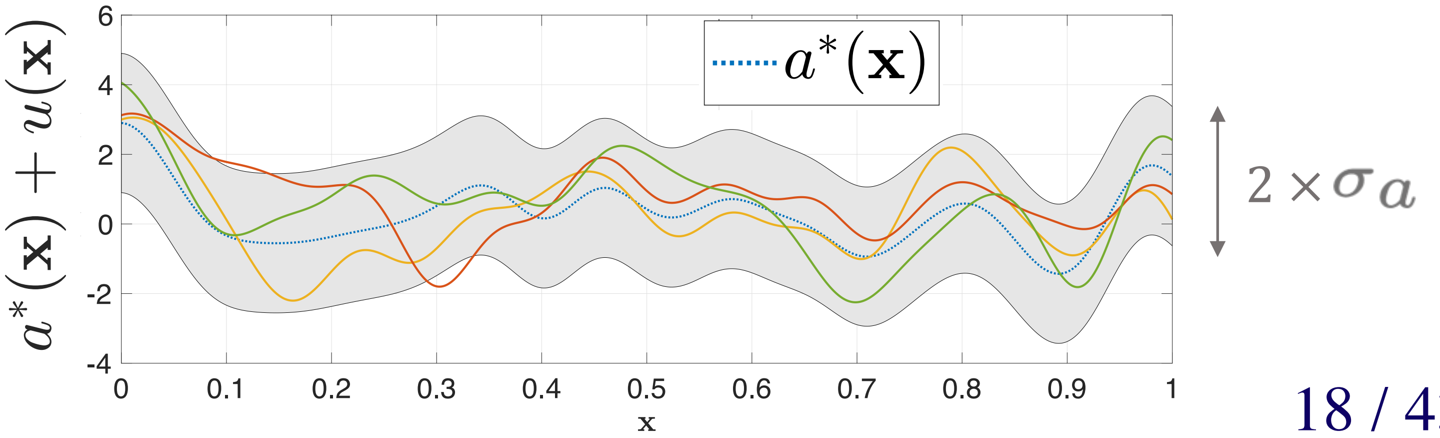
- Deterministic:

$$m^a(\mathbf{x}) = \underbrace{a^*(\mathbf{x})}_{\text{True spatial gain}} + \underbrace{b(\mathbf{x})}_{\text{Bias: systematic error}}$$



- Random:

$$\underbrace{u(\mathbf{x})}_{\text{Uncertainty}} \sim \mathcal{GP}(0, k^a(\mathbf{x}, \mathbf{x}'))$$



Effect of bias and uncertainty on the output SNR

Estimated spatial gain: $\hat{a}(\mathbf{x}) = a^*(\mathbf{x}) + \underbrace{b(\mathbf{x})}_{\text{Bias}} + \underbrace{u(\mathbf{x})}_{\text{Uncertainty}}$

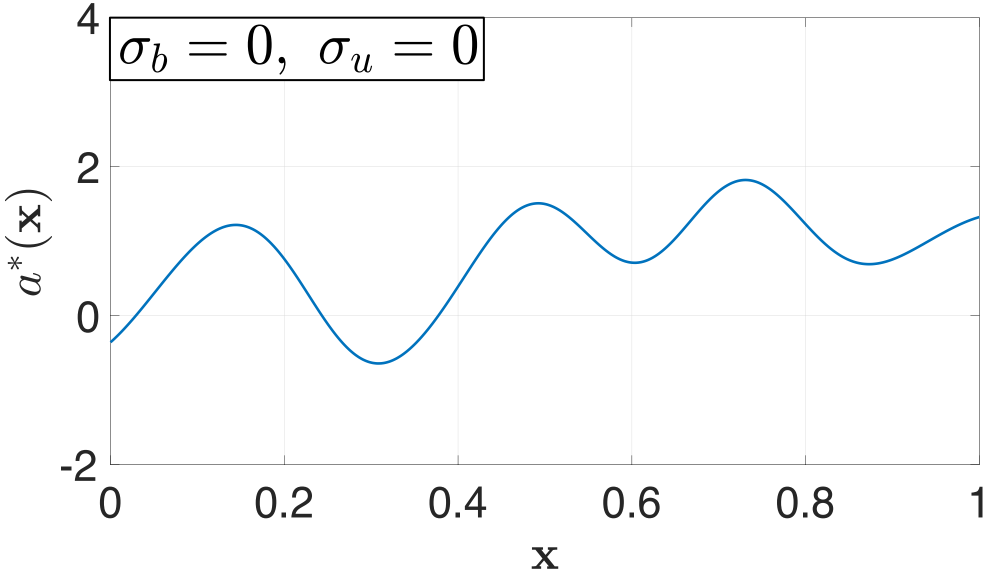
- Oracle: true SNR with the true values of the spatial gain :

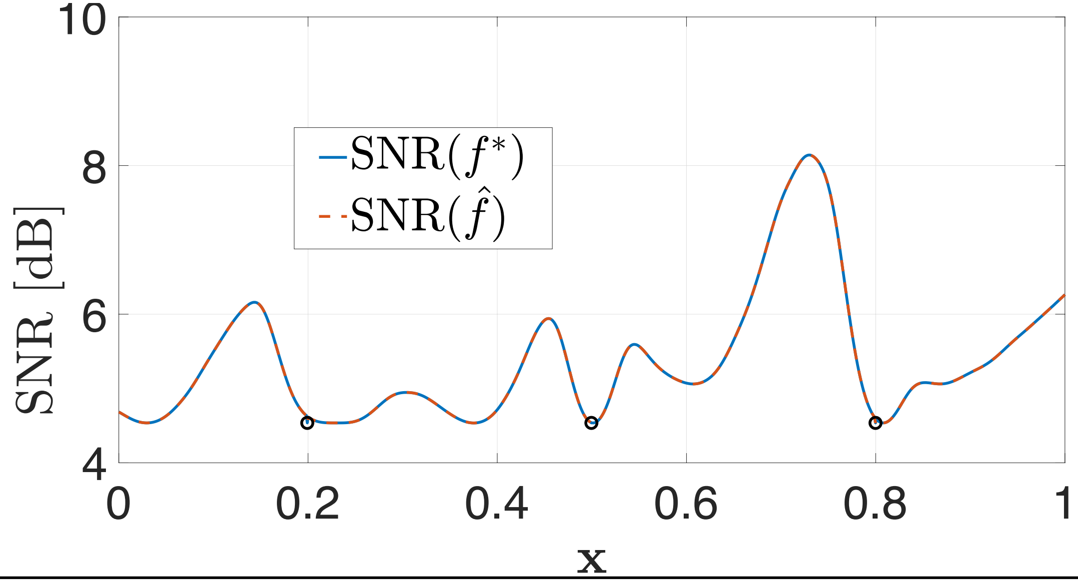
$$\boxed{\mathbf{f}^*(\mathbf{X}_\mathcal{M}) = \mathbf{C}^n(\mathbf{X}_\mathcal{M}, \mathbf{X}_\mathcal{M})^{-1}\mathbf{a}^*(\mathbf{X}_\mathcal{M})} \Longrightarrow \mathrm{SNR}(\mathbf{f}^*(\mathbf{X}_\mathcal{M})) = \frac{\mathbb{E}_t\left[\left(\mathbf{f}^*(\mathbf{X}_\mathcal{M})^T\mathbf{a}^*(\mathbf{X}_\mathcal{M})s(t)\right)^2\right]}{\mathbb{E}_t\left[\left(\mathbf{f}^*(\mathbf{X}_\mathcal{M})^T\mathbf{n}(\mathbf{X}_\mathcal{M}, t)\right)^2\right]}$$

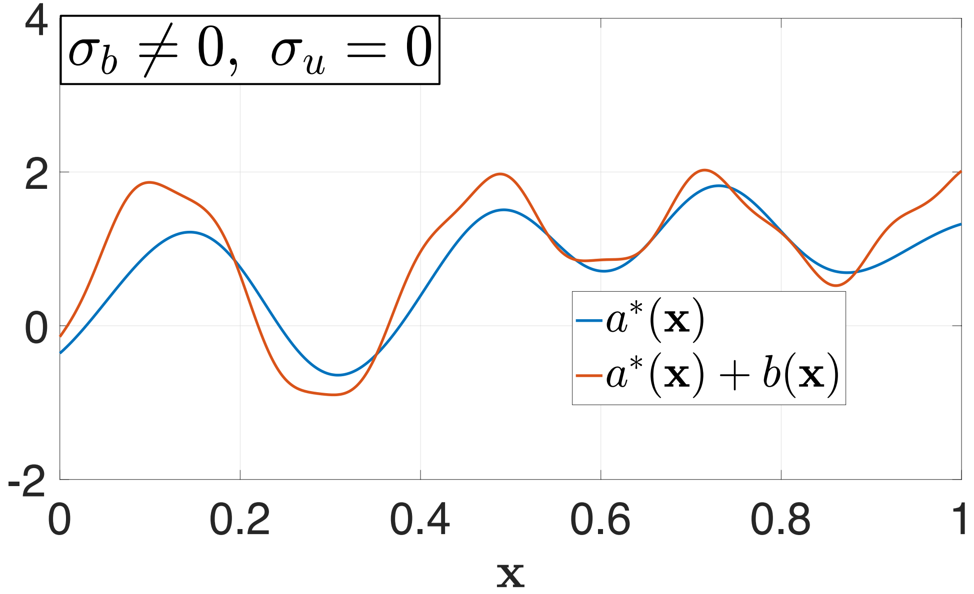
- Achieved SNR: true SNR with uncertainty on the spatial gain:

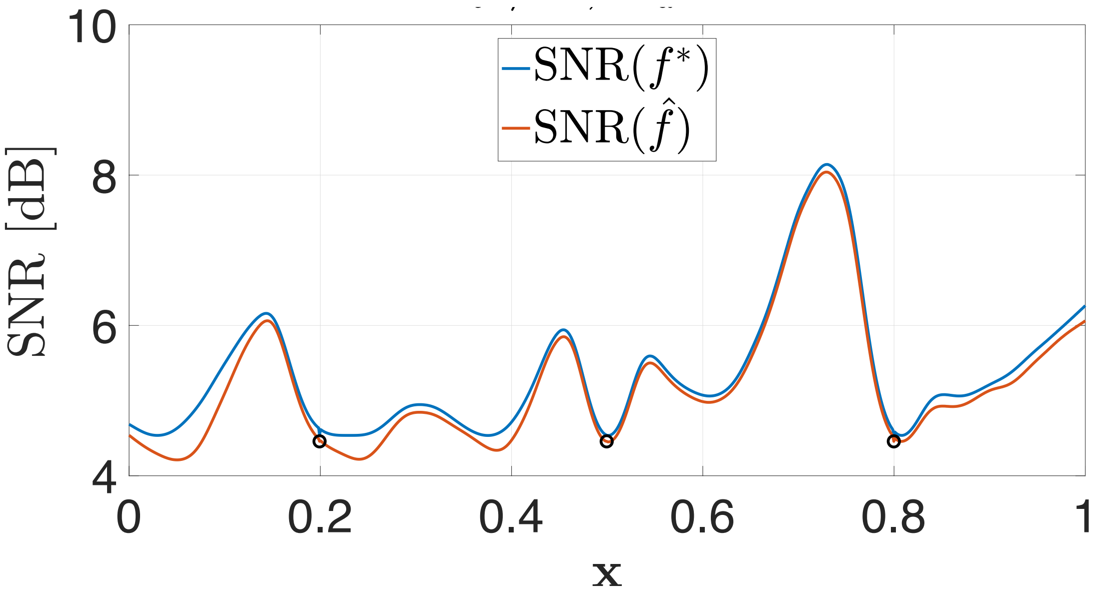
$$\boxed{\hat{\mathbf{f}}(\mathbf{X}_\mathcal{M}) = \mathbf{C}^n(\mathbf{X}_\mathcal{M}, \mathbf{X}_\mathcal{M})^{-1}\hat{\mathbf{a}}(\mathbf{X}_\mathcal{M})} \Longrightarrow \mathrm{SNR}(\hat{\mathbf{f}}(\mathbf{X}_\mathcal{M})) = \frac{\mathbb{E}_t\left[\left(\hat{\mathbf{f}}(\mathbf{X}_\mathcal{M})^T\mathbf{a}^*(\mathbf{X}_\mathcal{M})s(t)\right)^2\right]}{\mathbb{E}_t\left[\left(\hat{\mathbf{f}}(\mathbf{X}_\mathcal{M})^T\mathbf{n}(\mathbf{X}_\mathcal{M}, t)\right)^2\right]}$$

The effect of bias

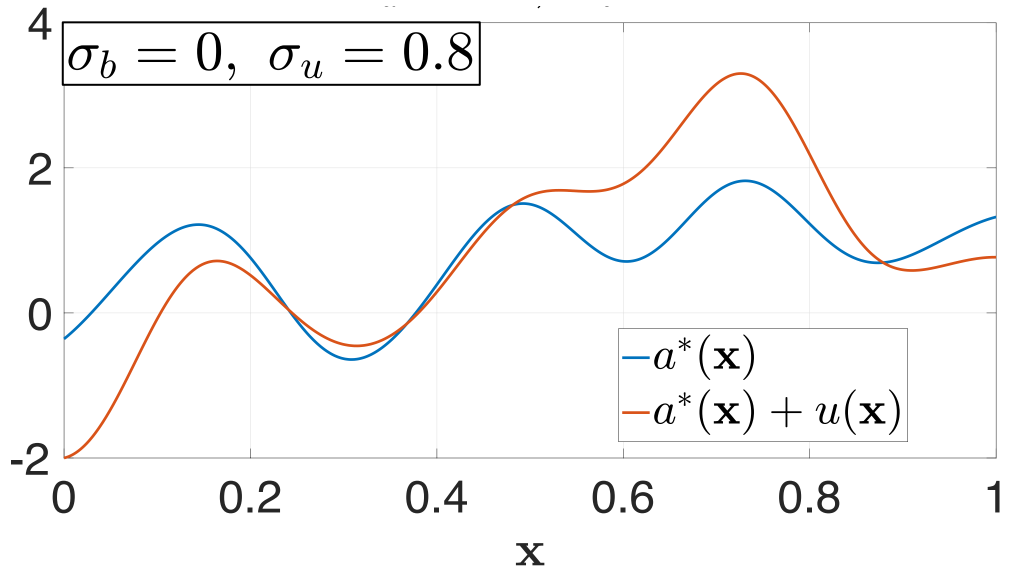


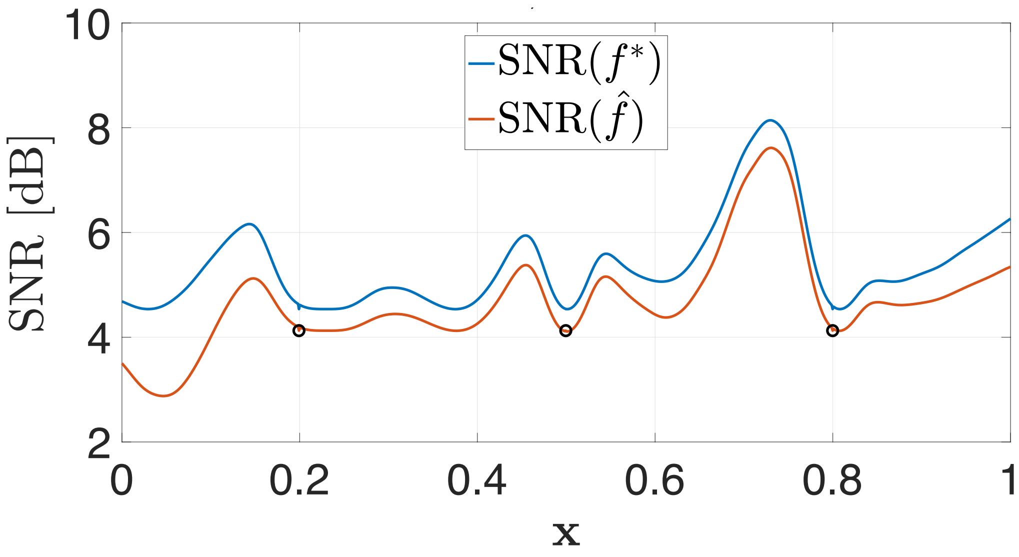


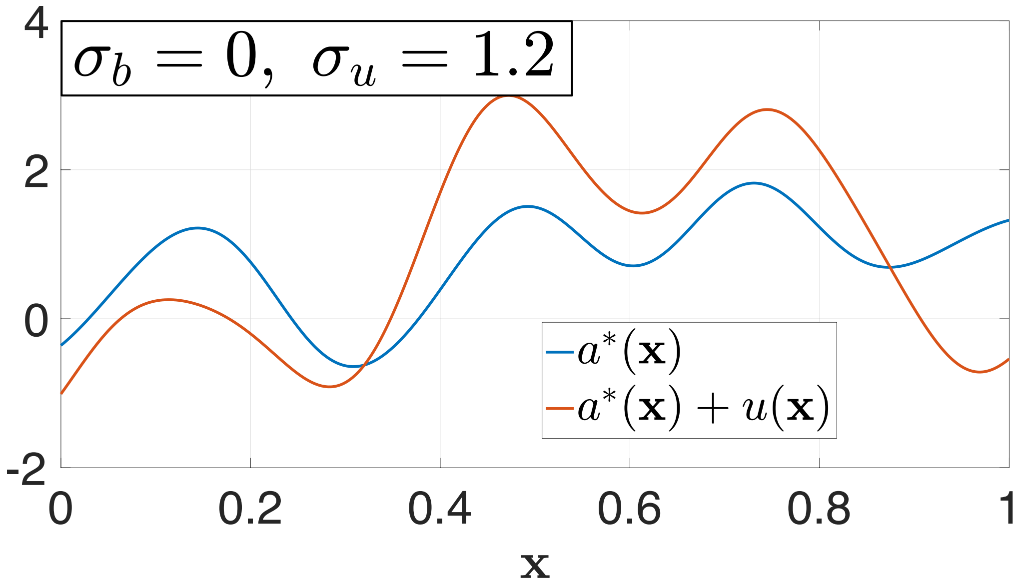


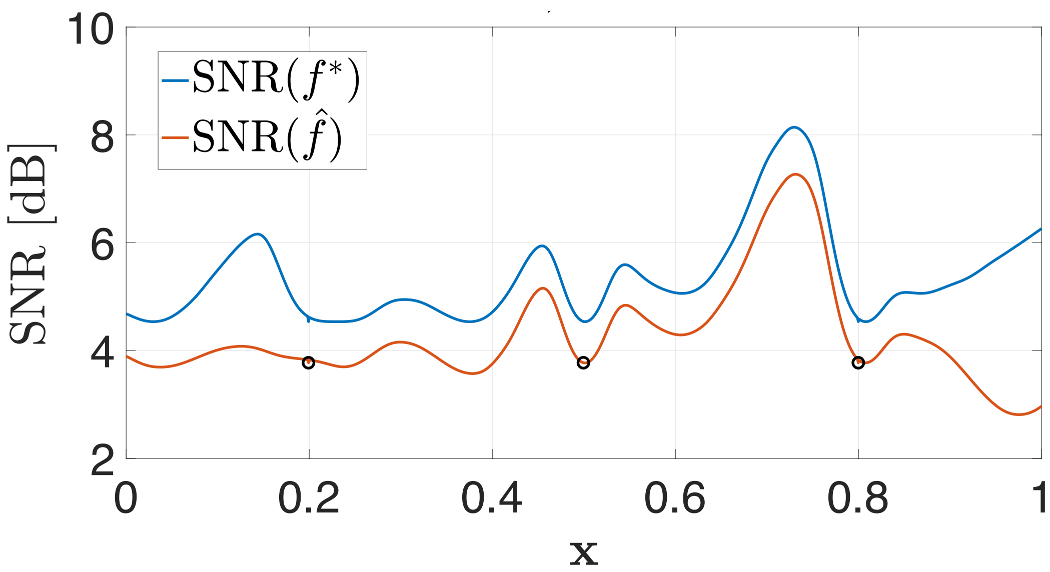


The effect of uncertainty









Numerical results

Simulation setup

One-dimension grid: $\mathbf{X}_{\mathcal{P}} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_P]^T$, in the normalized range $\mathbf{x}_i \in [0, 1]$

- The grid size P : depending on the smoothness of the signal

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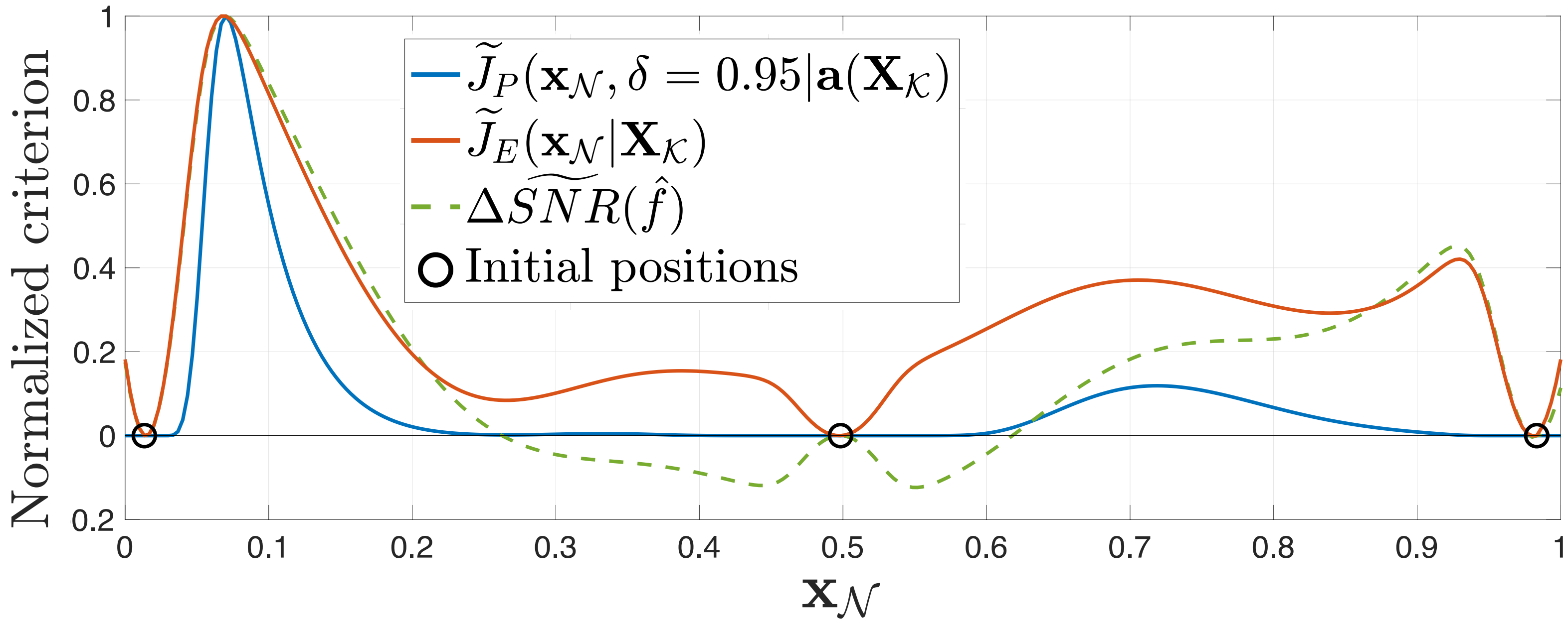
Prior knowledge:

$$\hat{a}(\mathbf{x}) = a^*(\mathbf{x}) + \underbrace{b(\mathbf{x})}_{\text{Bias}} + \underbrace{u(\mathbf{x})}_{\text{Uncertainty}}$$

- $a(\mathbf{x})$, $n(\mathbf{x})$, $u(\mathbf{x})$: GP models $\mathcal{GP}\big(m(\mathbf{x}), C(\mathbf{x}, \mathbf{x}')\big)$
- $b(\mathbf{x})$: drawn from GP
- $C(\mathbf{x}, \mathbf{x}') = \sigma^2 \exp(-(\mathbf{x} - \mathbf{x}')^2 / (2\rho^2))$

Notations (cont'd)



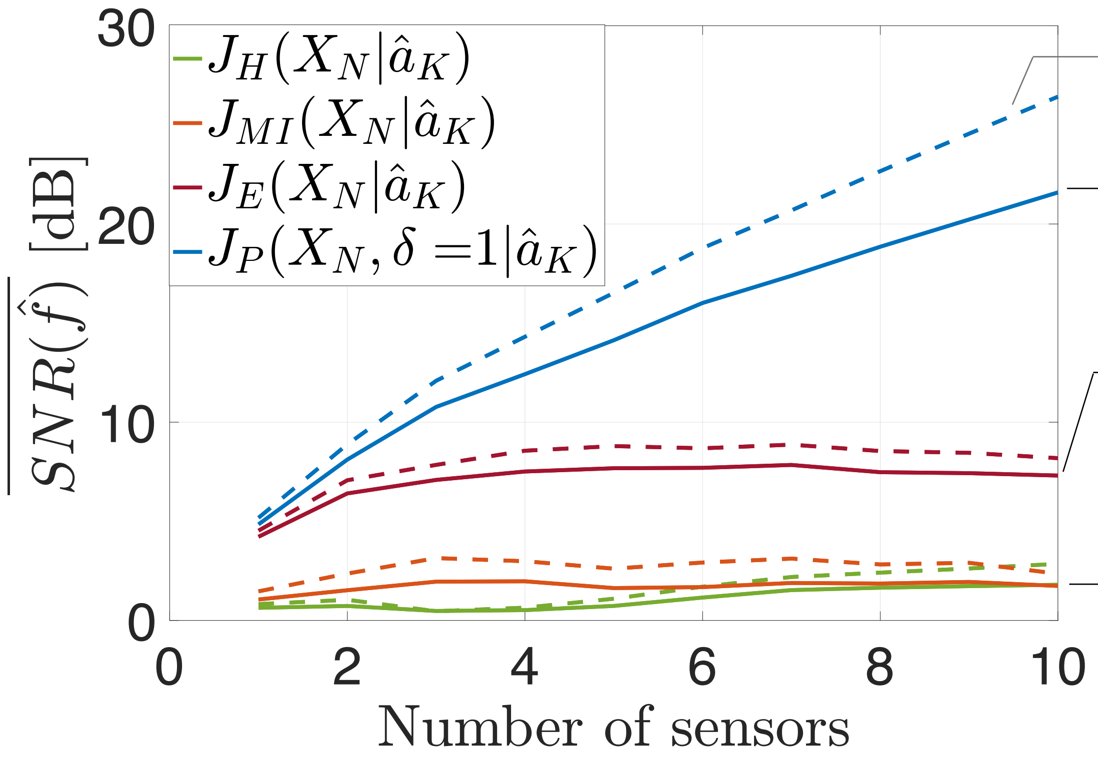
$$\Delta\widetilde{\mathrm{SNR}}(\hat{\boldsymbol{f}}(\mathbf{X}_\mathcal{M})) = \frac{\mathrm{SNR}(\hat{\boldsymbol{f}}(\mathbf{X}_\mathcal{M})) - \mathrm{SNR}(\hat{\boldsymbol{f}}(\mathbf{X}_\mathcal{K}))}{\mathrm{SNR}_{max}(\hat{\boldsymbol{f}}(\mathbf{X}_\mathcal{M})) - \mathrm{SNR}(\hat{\boldsymbol{f}}(\mathbf{X}_\mathcal{K}))} \Longrightarrow \boxed{\begin{array}{l} \max\left[\Delta\widetilde{SNR}(\hat{f}(\mathbf{X}_\mathcal{M}))\right] = 1 \\ \Delta\widetilde{SNR}(\hat{f}(\mathbf{X}_\mathcal{K})) = 0 \end{array}}$$

$$\theta = \widehat{\mathrm{SNR}}(\hat{\boldsymbol{f}}(\mathbf{X}_\mathcal{K})) + \delta\left[\widehat{\mathrm{SNR}}(\hat{\boldsymbol{f}}(\mathbf{X}_\mathcal{M})) - \widehat{\mathrm{SNR}}(\hat{\boldsymbol{f}}(\mathbf{X}_\mathcal{K}))\right]$$

Average SNR: $J_E(\mathbf{x}_\mathcal{N}|\mathbf{X}_\mathcal{K}) = \mathbb{E}\left\{\tilde{\mathbf{a}}(\mathbf{X}_\mathcal{M})^T \mathbf{C}^n(\mathbf{X}_\mathcal{M}, \mathbf{X}_\mathcal{M})^{-1}\tilde{\mathbf{a}}(\mathbf{X}_\mathcal{M}) \middle| \mathbf{X}_\mathcal{K}\right\}$

['*Optimal Sensor Placement for Signal Extraction*', F. Ghayem, B. Rivet, C. Jutten, R. C. Farias, ICASSP2019, Brighton, UK]

Performance for sequential approach



Dashed curves: results obtained by the true $\mathbf{a}(\mathbf{X}_{\mathcal{K}})$

Probabilistic criterion: J_P

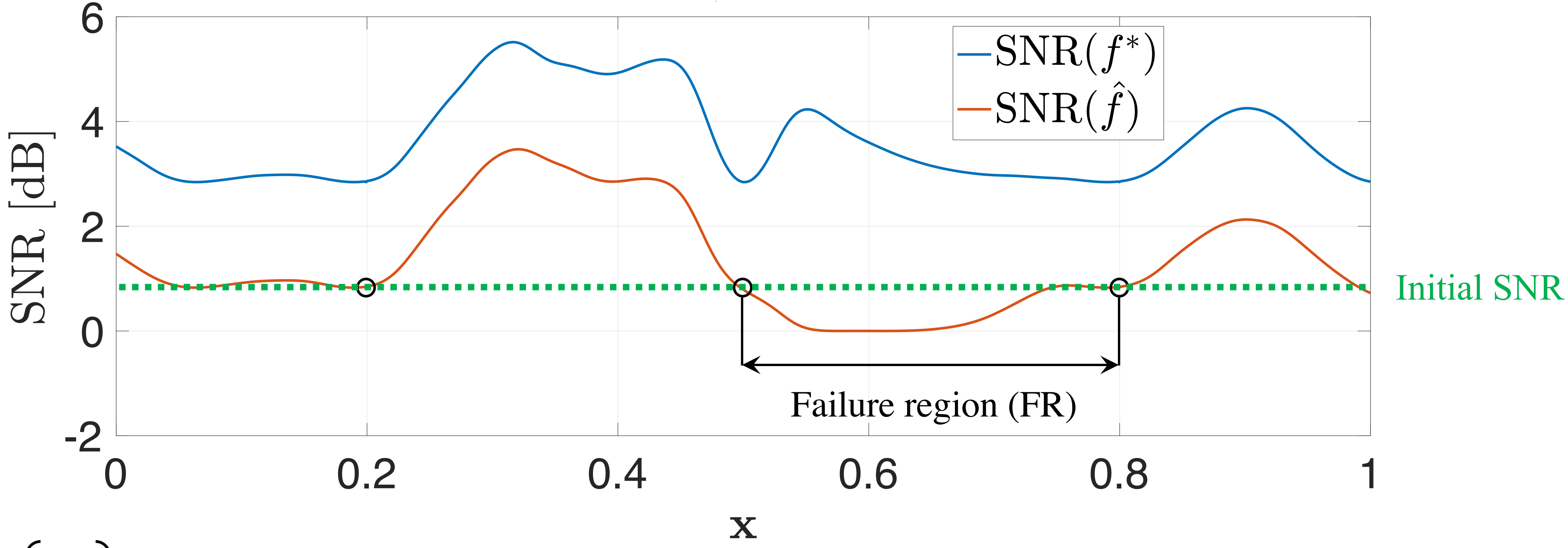
Average SNR: J_E

Classical kriging: $\begin{cases} J_{MI}: \text{mutual information} \\ J_H: \text{entropy} \end{cases}$

Results:

- J_P provides larger output SNR
- Uncertainty on $\mathbf{a}(\mathbf{X}_{\mathcal{K}})$ $\Longrightarrow$ Decreasing the SNR

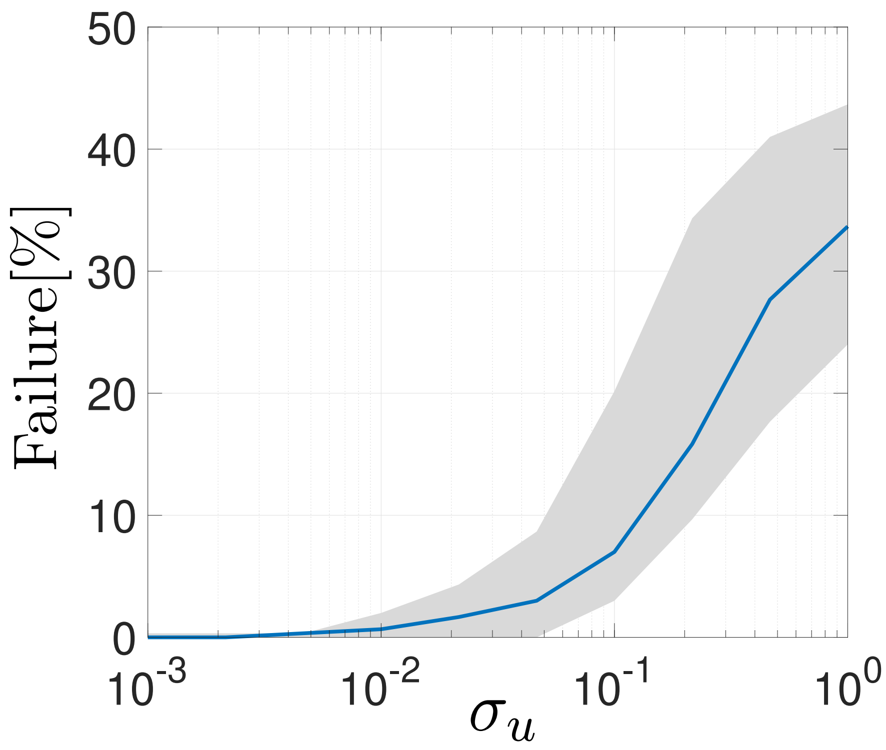
Failure[%]



$$\mathbf{X}_{\mathcal{M}} = \begin{bmatrix} \mathbf{X}_{\mathcal{K}} \\ \mathbf{X}_{\mathcal{N}} \end{bmatrix} \qquad \mathrm{SNR}(\hat{f}(\mathbf{X}_{\mathcal{M}})) < \mathrm{SNR}(\hat{f}(\mathbf{X}_{\mathcal{K}})) \Longrightarrow \text{Failure}$$

$$\boxed{\text{Failure}\,[\%] \triangleq \frac{\text{Size of the FR}}{\text{Total size of the spatial grid}} = \frac{N_{FR}}{N_P}}$$

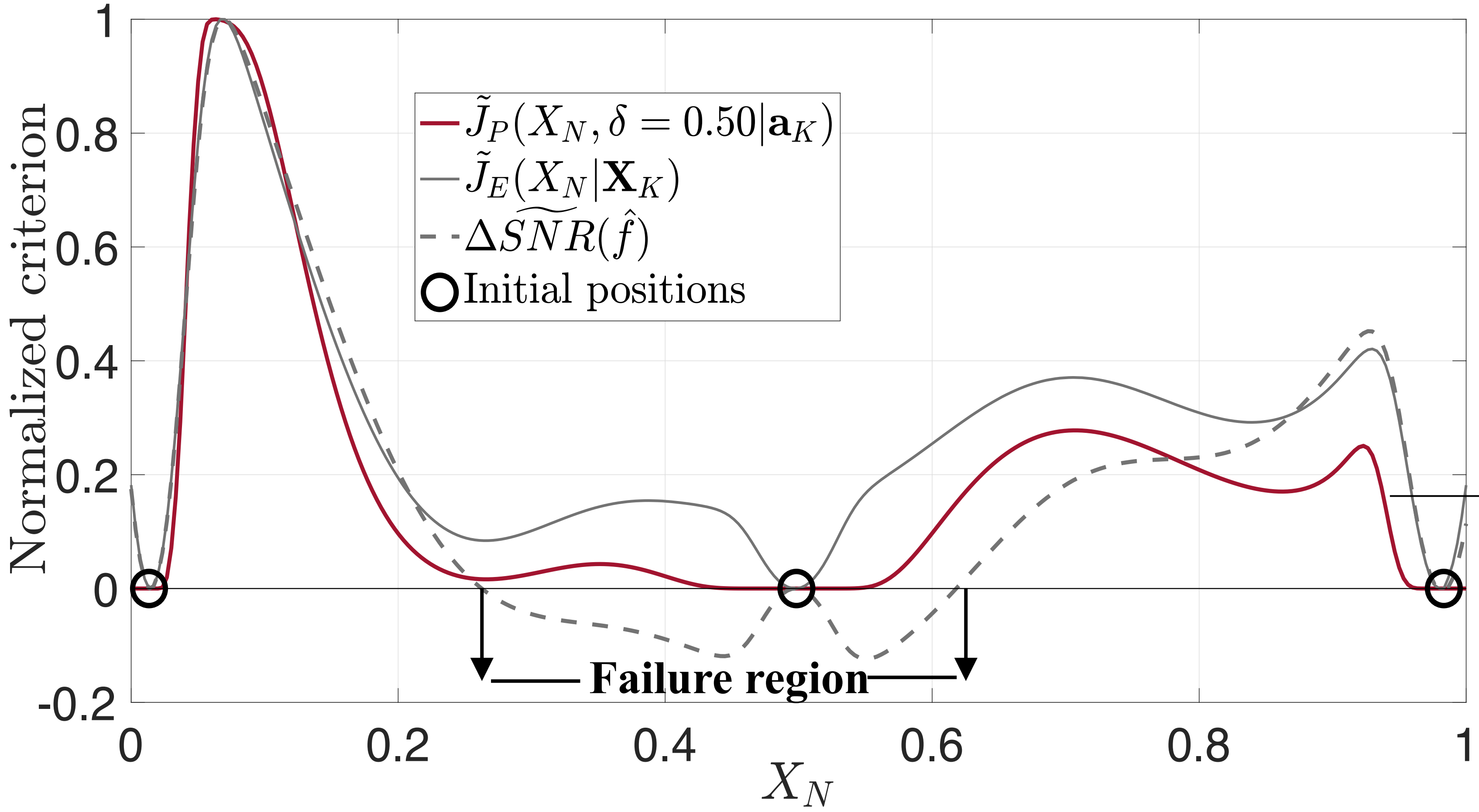
The effect of uncertainty on Failure[%]



Result:

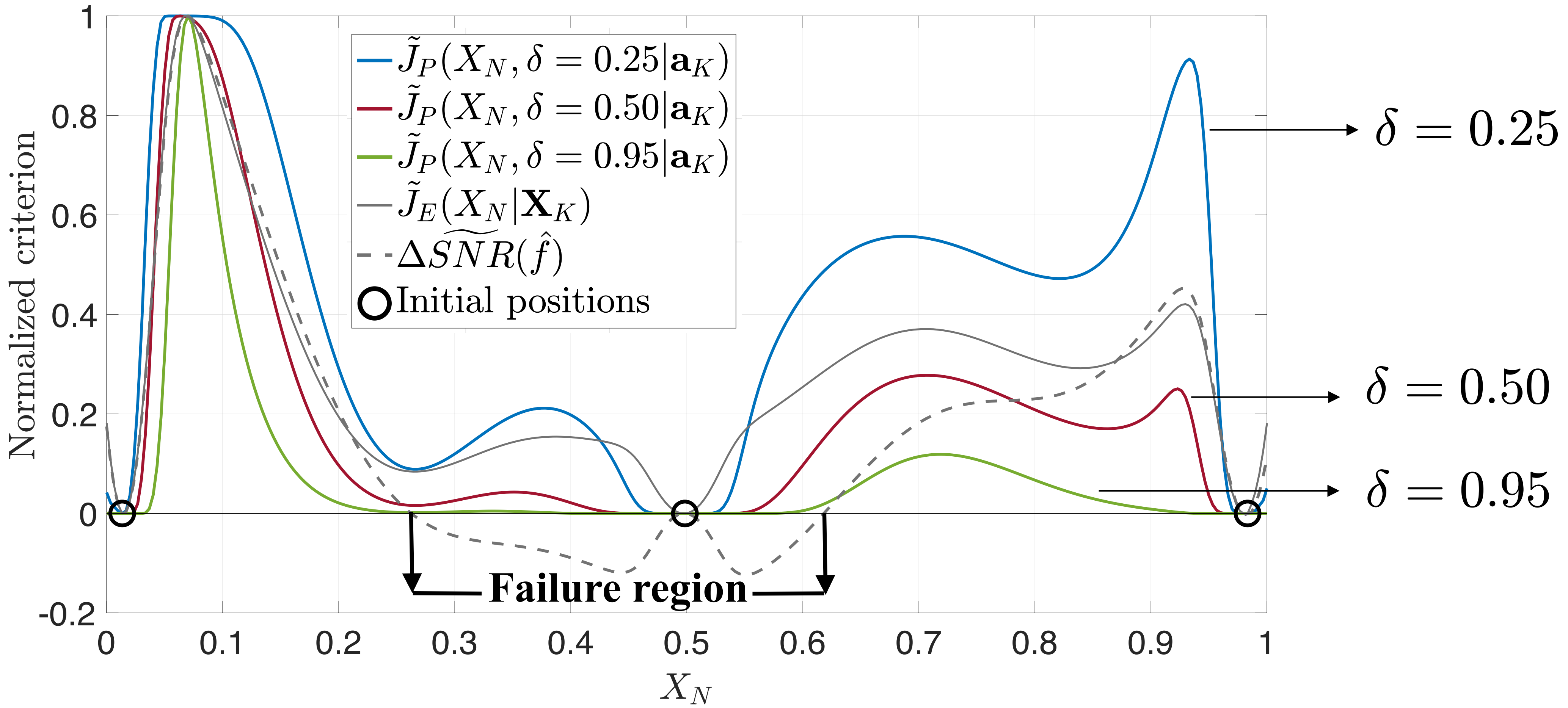
Large σ_u $\Longrightarrow$ Increase Failure[%]

Robustness: the effect of parameter δ on J_P



$\delta = 0.50$

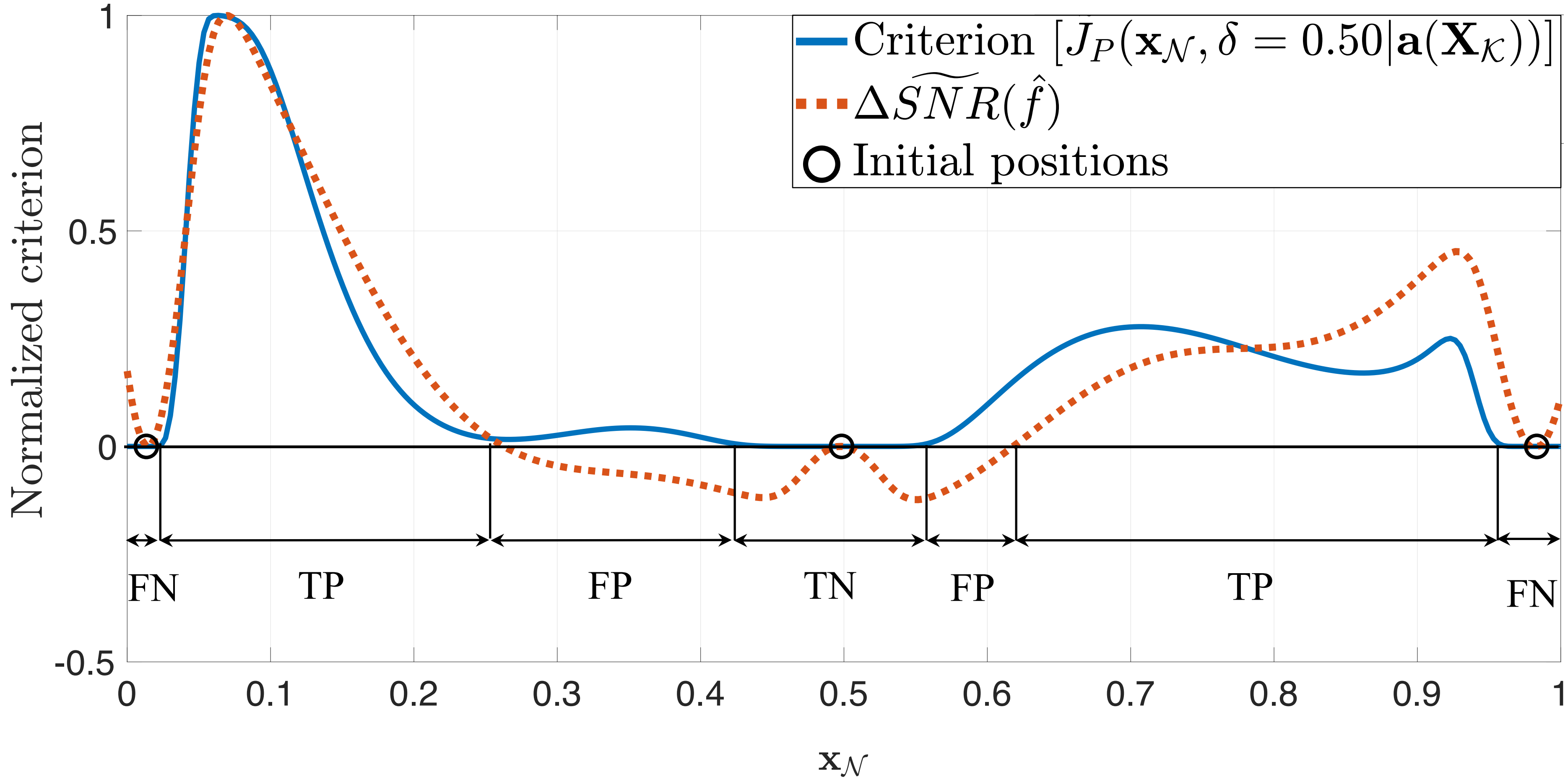
Robustness: the effect of parameter δ on J_P



Result:

Increasing δ $\Longrightarrow$ Increasing robustness against FR

How to measure robustness?

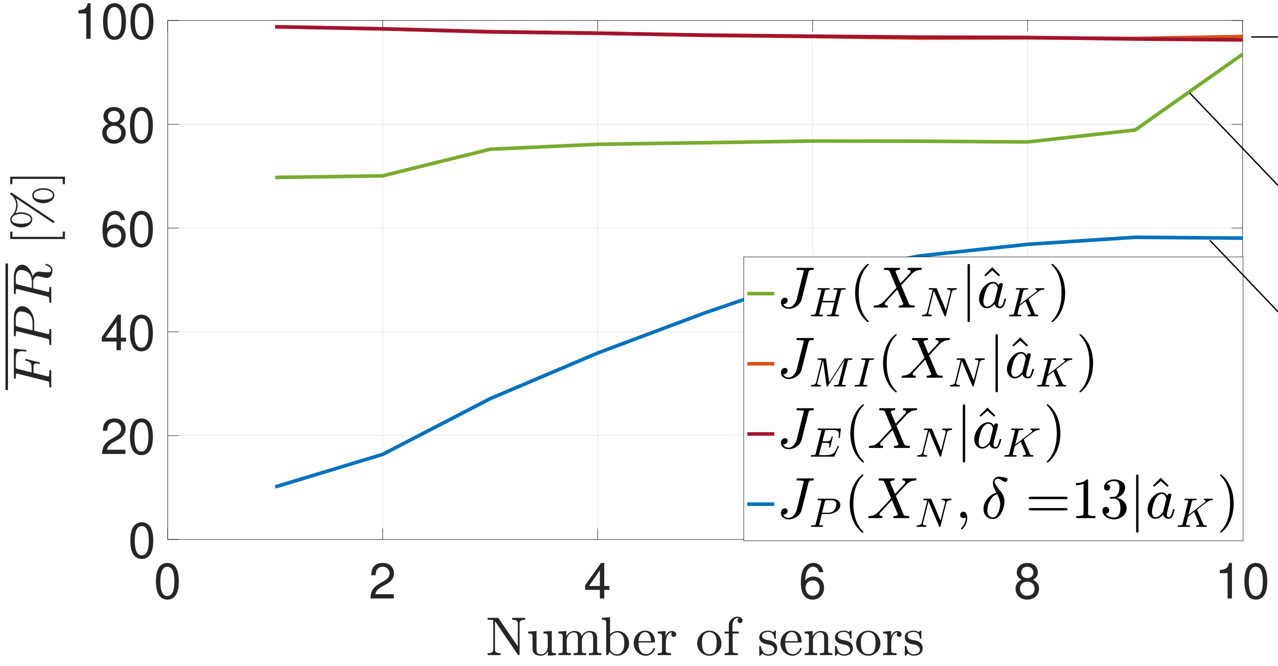


- TP: true positive
- TN: true negative
- FP: false positive
- FN: false negative

False positive rate:

$$FPR = \frac{false\ positive}{total\ number\ of\ negatives} = \frac{FP}{TN + FP}$$

Robustness

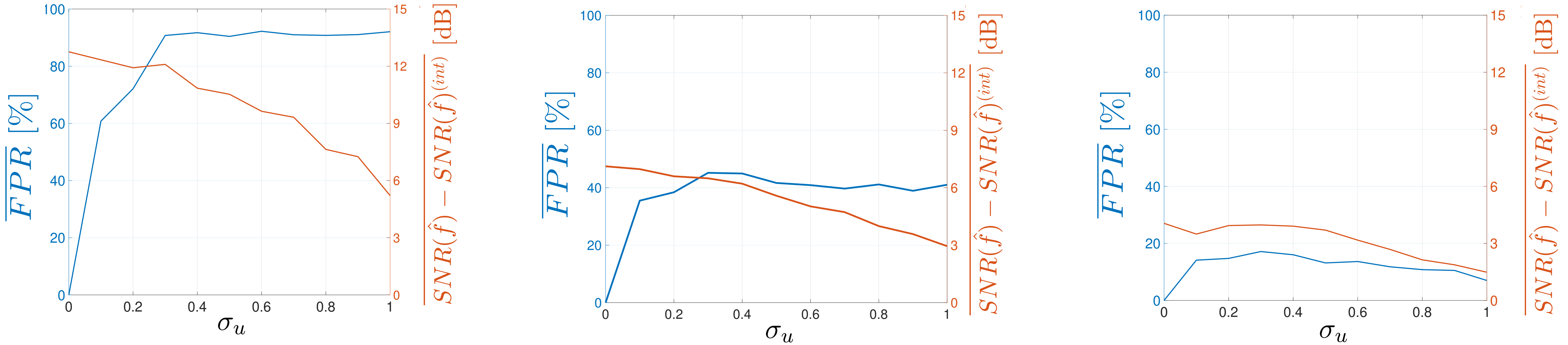


- Mutual information: J_{MI}
- Average SNR: J_E
- Entropy: J_H
- Probabilistic criterion: J_P

Result:

- J_P is more robust against the uncertainty of the spatial gain

Trade-off between robustness and average SNR maximization



(a) $\delta = 1$ (b) $\delta = 10$ (c) $\delta = 20$

Results:

- Increasing δ $\Longrightarrow$ $\begin{cases} \text{Decreasing} & \overline{FPR}[\%] \\ \text{Decreasing} & \overline{\Delta\text{SNR}(\hat{\boldsymbol{f}}(\mathbf{X}_{\mathcal{M}}))} \end{cases}$

Trade-off between
- increasing the SNR
- reducing FPR

Contributions

I. **Criterion:** Robust sensor placement for signal extraction

II. **Optimization:** Gradient-based algorithm with spatial regularization

How can we improve the greedy approach?

- Two limitations:
    1. Restricting sensor location on a predefined grid
    2. Suboptimal solution: greedy approach (previous sensors' locations are not modified.)

To be more accurate: Fine grid ⟹
- High computation cost
- Multiple closely spaced sensors

- Existing solutions *e.g.* branch-and-bound method:

High dimensions ⟹ High computational complexity

Our proposed method: a two-step method

- **Step1.** Greedy initialization
- **Step2.** Optimization: adjusting the sensor positions

Gradient-based algorithm with spatial regularization

- The average output SNR as the target function:

$$J_E(\mathbf{X}_\mathcal{M}) = \mathbf{m}^a(\mathbf{X}_\mathcal{M})^T \mathbf{C}^n(\mathbf{X}_\mathcal{M}, \mathbf{X}_\mathcal{M})^{-1} \mathbf{m}^a(\mathbf{X}_\mathcal{M}) + \mathrm{Tr}\left[\mathbf{C}^n(\mathbf{X}_\mathcal{M}, \mathbf{X}_\mathcal{M})^{-1} \mathbf{C}^a(\mathbf{X}_\mathcal{M}, \mathbf{X}_\mathcal{M})\right]$$

Spatial regularization
to avoid too close sensors

$$\min_{\mathbf{X}_\mathcal{M}} \; -J_E(\mathbf{X}_\mathcal{M}) \qquad \text{s.t.} \quad \begin{cases} \|\mathbf{D}\mathbf{X}_\mathcal{M}\|_2^2 \geq \epsilon \\ 0 \leq \mathbf{x}_i \leq 1, \quad i \in \{1, 2, \ldots, M\} \end{cases}$$

- $\|\mathbf{D}\mathbf{X}_\mathcal{M}\|_2^2 = \sum_{i=1}^{M} \sum_{j>i}^{M} |\mathbf{x}_i - \mathbf{x}_j|^2$: sum of the squared distances between each pair of sensors

[1] F. Ghayem, et al., *'Gradient-based algorithm with spatial regularization for optimal sensor placement'*, ICASSP'20, Spain

Gradient-based algorithm with spatial regularization

Auxiliary variable: $\mathbf{z}_M = \mathbf{D}\mathbf{x}_M$

$$\min_{\mathbf{x}_M,\mathbf{z}_M} -J(\mathbf{x}_M) \text{ s.t. } \begin{cases} \boxed{\mathbf{z}_M \in \mathcal{A}_\epsilon,} \\ \boxed{\mathbf{z}_M = \mathbf{D}\mathbf{x}_M,} \\ 0 \leq x_i \leq 1 \;\; i \in \{1,\dots,M\} \end{cases}$$

$$\mathcal{A}_\epsilon = \left\{ \mathbf{z}_M \in \mathbb{R}^M \;\middle|\; \|\mathbf{z}_M\|_2^2 \geq \epsilon \right\}$$

- Penalty method:

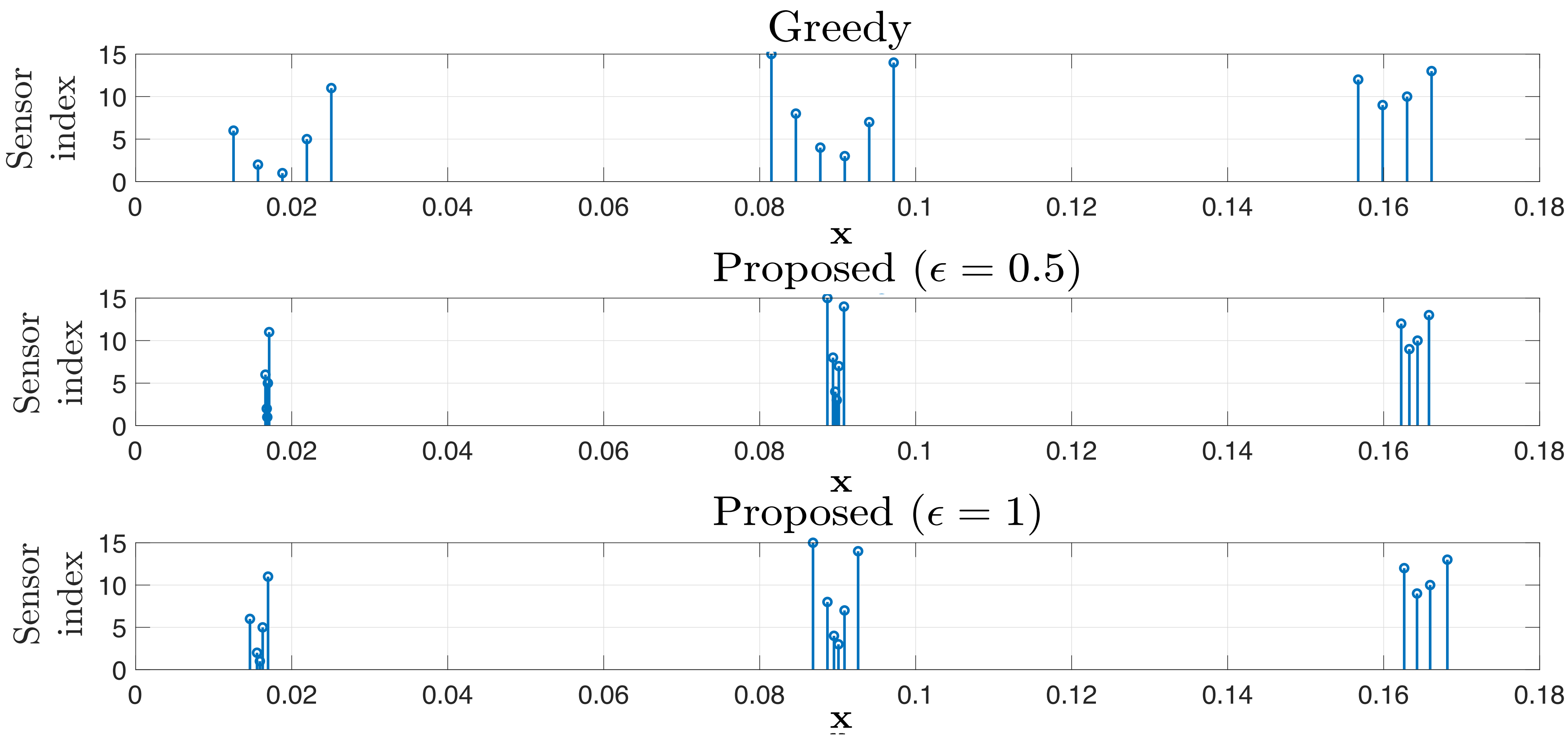
$$\min_{\mathbf{x}_M,\mathbf{z}_M \in \mathcal{A}_\epsilon} \left\{ -J(\mathbf{X}_M) + \boxed{\frac{1}{2\alpha}\|\mathbf{z}_M - \mathbf{D}\mathbf{x}_M\|_2^2} \right\}$$
$$\text{s.t. } \; 0 \leq x_i \leq 1 \;\; i \in \{1,\dots,M\}.$$

- Alternating minimization

Initialization with the greedy approach

Numerical results

Regularizing sensors distances



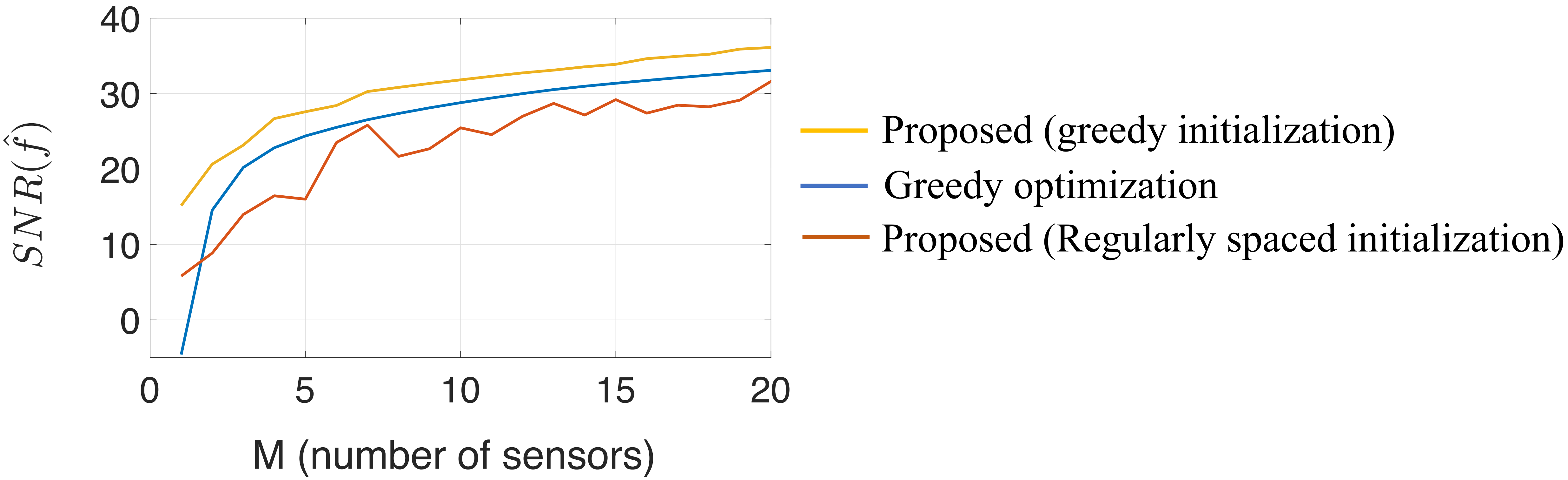
SNR: 29 dB
(Initial SNR)

SNR: 32 dB

SNR: 31 dB

Result: Increasing ϵ $\Longrightarrow$ Increasing the average distance between the sensors (with a slightly decrease of the output SNR)

Influence of the initialization



Results:

- Proposed optimization algorithm improves the SNR compared to the greedy approach.
- Greedy initialization: higher SNR than regularly-spaced initialization

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Conclusions & Perspectives

Conclusions

Problem statement

- Problem of optimal sensor placement
- Limited number of sensors
- Source signal extraction
- Measurements: linear instantaneous model
- Targeting the signal to noise ratio (SNR)
- Linear source extraction & GP assumption

Contributions

Criterion I: $J_E(\mathbf{X}_\mathcal{M}) = \mathbb{E}\Big\{w(\mathbf{X}_\mathcal{M})\Big\}$

Criterion II: $J_P(\mathbf{X}_\mathcal{M}, \theta) = Pr\big(w(\mathbf{X}_\mathcal{M}) > \theta\big)$

Algorithm I: Sequential approach

Algorithm II: Gradient-based approach

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Conclusions (cont'd)

Criterion I: $J_E(\mathbf{X}_{\mathcal{M}}) = \mathbb{E}\left\{w(\mathbf{X}_{\mathcal{M}})\right\}$

- Targeting the average SNR
- Closed-form expression
- Superiority to the classical kriging

Algorithm I: Greedy & sequential approaches

- Discrete optimization (combinatorial search)
- Sequentially adding the new $N < M$ sensors
- Updating the estimation of the spatial gain

Criterion II: $J_P(\mathbf{X}_{\mathcal{M}}, \theta) = Pr\left(w(\mathbf{X}_{\mathcal{M}}) > \theta\right)$

- Probabilistic criterion
- Distribution of the SNR
- Robust against the spatial gain uncertainty
- Trade-off: robustness & SNR improvement

Algorithm II: Gradient-based optimization

- Initialization with greedy approach
- Adjusting all sensors' locations at once
- Continuous space optimization
- Spatial constraint to control sensors' distances

Perspectives

Short-term

- Noise uncertainty:
  - ✓ pdf of the SNR based on the pdf of the spatial gain and the noise: (Wishart distribution)
- Estimation of the GP parameters: Bayesian inference
- Test the proposed methods in 2-D and 3-D settings

Long-term

- Multiple source extraction: BSS techniques
- Trade-off between the SNR improvement and the complexity: Akaike information criterion
- Dynamic design *e.g.* real-time applications, mobile source
- Acoustic signals: convolutive mixture model

Thank you